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by

John Paul Aucella

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ANALYSIS OF A SYSTEM RELIABILITY LOWER CONFIDENCE LIMIT ASSUMING GAMMA AND TRUNCATED NORMAL FAILURE DISTRIBUTIONS

by

John Paul Aucella
Lieutenant, United States Navy
B.S., United States Naval Academy, 1963

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

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ABSTRACT

The accuracy of the lower confidence limit procedure in NAVWEPS OD 29304 is analyzed when the failure distributions of the components in the series system are either gamma or truncated normal. Several representations of the accuracy are supplied.

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CHAPTER I

INTRODUCTION

The statistical model developed and stated in NAVWEPS OD 29304 "Guide Manual For Reliability Measurement Program" was developed for the Department of the Navy, Special Projects Office. It contains very versatile procedures for obtaining both system reliability point estimates and system reliability lower confidence limits using a variety of test data on component parts, subassemblies, assemblies, and subsystems.

This document makes the following assumptions on the components of the series system:

(1) Constant failure rate
(2) Additivity of stress effects
(3) Independence of part failure
(4) Failure rate constancy

The purpose of this thesis is to check the accuracy of this statistical model without the assumption of a constant failure rate. The model will be checked for a system composed of four components in series, using various sample sizes, mission times, and combinations of the gamma and truncated normal failure distributions. The statistical model will be analyzed by computer simulation using a Monte Carlo method to generate the gamma and truncated normal test times. The parameters of the distributions were selected so that the failure rate curves would be representative of those curves used by contractors or that the system reliability was a preselected value.

CHAPTER II

THE STATISTICAL MODEL

Section three of reference 1 contains a detailed explanation of the statistical model which is being analyzed. The following definitions are applicable parts of the model used in the computer simulation for the lower confidence limit of a series system with four components. For a given sample size, N_i, $i = 1,2,3,4$ and a predetermined test time, T_{oi}, $i = 1,2,3,4$ a best estimate of the failure rate for component i is given by:

$$\hat{\lambda}_i = \frac{f_i}{S_i} \frac{2N_i}{2N_i + 1} \tag{1}$$

Where $S_i = \sum_{j=1}^{N_i} T_{ij}$, is the sum of all test times accumulated on the N components of type i, and f_i is the number of failure times which are less than T_{oi}. From these best estimates of failure rates, the system reliability estimates are given by:

$$\hat{R}_s = e^{-\hat{\lambda}_s} \tag{2}$$

where $$\hat{\lambda}_s = \sum_{i=1}^{k} \hat{\lambda}_i \tag{3}$$

The model then uses the following formulae in order to compute the lower confidence limit on system reliability:

$$\hat{\lambda}_u = \frac{2\hat{\lambda}_s + (K\beta)^2 + \sqrt{4\hat{\lambda}_s(K\beta)^2\hat{C} + (K\beta)^4\hat{C}^2}}{2} \tag{4a}$$

$$\text{if} \quad \sum_{i=1}^{k} f_i > 0$$

$$\hat{\lambda}_u = \frac{(K\beta)^2}{n} \sum_{i=1}^{k} \frac{1}{S_i} \tag{4b}$$

$$\text{if} \quad \sum_{i=1}^{k} f_i = 0$$

where n is the number of tests.

Here $\hat{\lambda}_u$ is the upper confidence limit of the failure rate for a system with k units in series, and K is a percentage point of the normal distribution. β is a correction factor for small values of $\hat{\lambda}_i$ and $\hat{c} = \sum_{i=1}^{k} \frac{\hat{\lambda}_i}{S_i} / \hat{\lambda}_s$. All times are in mission units.

Reference 1 then defines $\hat{R}_{s,L(\alpha)}$ to be the lower confidence limit on system reliability:

$$\hat{R}_{s,L(\alpha)} = e^{-\hat{\lambda}_u} \tag{5}$$

It must be noted that since equation (2) is only a best estimate for the failure rate, the statistical model explained serves only as an approximation for system reliability.

CHAPTER III

SIMULATION PROCEDURE

The system to be simulated will consist of four components in a logical series, with a system reliability, R_s, which may be expressed as:

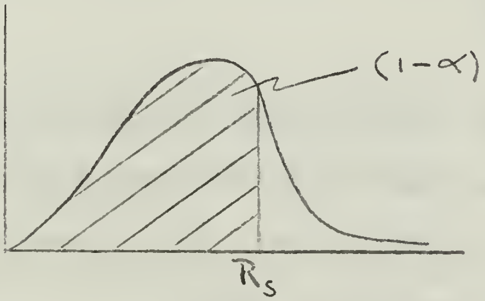
$$R_s = \prod_{i=1}^{4} R_i$$

where $R_i = P(T > 1)$ is the true reliability of the i^{th} component at mission time one. It is desired to find a lower $100(1-\alpha)\%$ confidence limit for R_s, namely the statistic $\hat{R}_{s,L(\alpha)}$. If $\hat{R}_{s,L(\alpha)}$ is in fact the exact $100(1-\alpha)\%$ lower confidence limit for R_s, then:

$$P[R_s > \hat{R}_{s,L(\alpha)}] = 1 - \alpha \qquad (6)$$

holds.

From equation (6), R_s should be the $(1-\alpha)$ percentile point of the simulated distribution of $\hat{R}_{s,L(\alpha)}$.



For the investigation α will be chosen as .20 and $A_{(1-\alpha)}$ will be defined as the $(1-\alpha)$ percentile point of the distribution of $\hat{R}_{s,L(\alpha)}$. In order to investigate the distribution of the random variable, a simulation by digital computer is used to generate the values needed. Five hundred values of $\hat{R}_{s,L(\alpha)}$ will be calculated and ordered such that:

$$\hat{R}_{s,L(\alpha)}(1) < \hat{R}_{s,L(\alpha)}(2) < \hat{R}_{s,L(\alpha)}(3) < \ . \ . \ . \ \hat{R}_{s,L(\alpha)}(500)$$

Since the value of α chosen for the simulation is .20, the value of $\hat{R}_{s,L(\alpha)}$ will be found such that 20% of the ordered values are above it. Thus the 80th percentile of the distribution of $\hat{R}_{s,L(\alpha)}$, $A_{(1-\alpha)}$, is found and should equal R_s. The measure of accuracy will therefore be the value $\left|R_s - A_{(1-\alpha)}\right|$. The accuracy of the method will be checked for different combinations of the failure rate distributions among the four components. The sample size, N_1, to be tested will consist of 50, 10, and 500 units. The mission times for each component, T_{oi}, will be:

(1) .5 mission units

(2) 5.0 mission units

(3) T_{oi} such that $-\frac{1}{T_{oi}}\ln[R(T_{oi})] = Z(1)$ (7)

where $Z(1) = \frac{f(1)}{R(1)}$ (8)

The third test was selected as an ad hoc rule because of the intuitive feeling that this time would give a good measure of accuracy to the procedure. The distribution of $\hat{R}_{s,L(\alpha)}$ will therefore be dependent upon:

(1) α

(2) $N_i \quad i = 1,2,3,4$

(3) $T_{oi} \quad i = 1,2,3,4$

(4) $R_i \quad i = 1,2,3,4$

The mean and the standard deviation of the distribution of $\hat{R}_{s,L(\alpha)}$ will also be calculated in order to better evaluate the accuracy of the model. The procedure for the computer simulation of the model is:

(1) Generate random times, $T'_{ij} \quad i = 1,2,3,4 \quad j = 1,2,\text{---}N$ for each of the four components in series.

$$\begin{array}{cccccc} T'_{11} & T'_{12} & T'_{13} & \cdot & \cdot & \cdot & T'_{1N} \\ T'_{21} & T'_{22} & T'_{23} & \cdot & \cdot & \cdot & T'_{2N} \\ T'_{31} & T'_{32} & T'_{33} & \cdot & \cdot & \cdot & T'_{3N} \\ T'_{41} & T'_{42} & T'_{43} & \cdot & \cdot & \cdot & T'_{4N} \end{array}$$

(2) Test these times against the selected mission time, T_{oi}, $i = 1,2,3,4$ and if $T'_{ij} < T_{oi}$, record a failure or if $T'_{ij} \geq T_{oi}$, record a mission success and then set

$$T_{ij} = \left\{ \begin{array}{ll} T'_{ij} & \text{if } T'_{ij} < T_{oi} \\ T_{oi} & \text{if } T'_{ij} \geq T_{oi} \end{array} \right\}$$

(3) From the equations (1) - (6) outlined in Chapter II, calculate the resulting values of $\hat{R}_{s,L(\alpha)}$ and $\hat{R}_s$.

(4) Repeat the above steps five hundred times.

(5) Order the computed values of $\hat{R}_{s,L(\alpha)}$.

(6) Calculate the mean and standard deviation of the resulting distribution and find the 80th percentile point, $A_{(1-\alpha)}$.

The complete computer simulation is found in Appendix C. The procedure used is shown above and comment statements have been used to aid in following the steps of the program.

CHAPTER IV

RESULTS

For the procedure stated in Chapter III, the failure times generated will be either gamma distributed or truncated normal distributed. The parameters of the distributions were chosen so that the reliability of the component was a preselected value or the failure rate curves would be representative of those curves used by contractors. The following cases were simulated:

CASE	COMPONENT	DISTRIBUTION	PARAMETERS	R_i
I	1,2 3,4	gamma tr. normal	r=1.5, s=.036 $\mu=11.28$, $\sigma=4$	.9950 .9950
II	1,2 3,4	gamma tr. normal	r=1.5, s=.036 $\mu=26$, $\sigma=10$	.9950 .9984
III	1,2 3,4	gamma tr. normal	r=1.5, s=.036 $\mu=-30$, $\sigma=45$	.9950 .9707
IV	1,2,3,4	gamma	r=1.5, s=.036	.9950
V	1,2,3,4	tr. normal	$\mu=8$, $\sigma=40$	.9810
VI	1,2,3,4	tr. normal	$\mu=20$, $\sigma=40$	.9850
VII	1,2,3,4	tr. normal	$\mu=-30$, $\sigma=45$	.9707

Each case is specified by the type of distribution and parameters applied to the four components in series. The results in Table I are for the first three cases using all possible combinations of sample sizes and test times. The results of cases IV, V, VI, and VII are given in Table II. These cases are different from the first three in that miscellaneous test times and sample sizes are used in the simulation.

The first five columns in the tables are self explanatory and the last is the value of TT which is represented by the formula $TT = \sum_{i=1}^{4} N_i \hat{\lambda}_i T_{oi}$. It is a measure of the amount of testing required, relative to the component unreliability.

Appendix A contains discussions of the gamma and truncated normal failure distributions. Appendix B contains the computation of the ad hoc test times used in Table I.

TABLE I

ACCURACY OF COMPUTER SIMULATION FOR CASES I, II, AND III

CASE I R_s = .9801

		$A_{(1-\alpha)}$	$\hat{\hat{R}}_{s,L(\alpha)}$		$\hat{R}_s$		accuracy	TT
			mean	std.dev	mean	std.dev		
Ni = 50 Toi = .5	i = 1,2,3,4	.9376	.9239	.0243	.9894	.0189	.0425	.5
Ni = 100 Toi = .5	i = 1,2,3,4	.9683	.9519	.0193	.9875	.0149	.0118	1.0
Ni = 500 Toi = .5	i = 1,2,3,4	.9881	.9792	.0080	.9888	.0065	.0080	2.5
Ni = 50 Toi = 5.	i = 1,2,3,4	.9549	.9428	.0136	.9572	.0122	.0252	5.0
Ni = 100 Toi = 5.	i = 1,2,3,4	.9573	.9478	.0094	.9548	.0088	.0228	10.0
Ni = 500 Toi = 5.	i = 1,2,3,4	.9575	.9539	.0041	.9574	.0039	.0226	25.0
Ni = 50 Toi=2.4 Toi=1.8	i = 1,2,3,4 i = 1,2 i = 3,4	.9753	.9581	.0173	.9788	.0141	.0048	2.1
Ni = 100 Toi=2.4 Toi=1.8	i = 1,2,3,4 i = 1,2 i = 3,4	.9771	.9653	.0123	.9780	.0104	.0030	4.2
Ni = 500 Toi=2.4 Toi=1.8	i = 1,2,3,4 i = 1,2 i = 3,4	.9781	.9740	.0047	.9785	.0044	.0020	21.0

TABLE I (Continued)

CASE II R_s = .9868

	$A_{(1-\alpha)}$	$\hat{R}_{s,L(\alpha)}$		$\hat{R}_s$		accuracy	TT
		mean	std.dev	mean	std.dev		
Ni = 50 i = 1,2,3,4 Toi = .5	.9376	.9257	.0240	.9908	.0188	.0492	.33
Ni = 100 i = 1,2,3,4 Toi = .5	.9683	.9552	.0176	.9901	.0136	.0185	.66
Ni = 500 i = 1,2,3,4 Toi = .5	.9881	.9803	.0079	.9902	.0065	.0013	3.3
Ni = 50 i = 1,2,3,4 Toi = 5.	.9733	.9635	.0110	.9756	.0095	.0135	3.3
Ni = 100 i = 1,2,3,4 Toi = 5.	.9728	.9665	.0078	.9744	.0070	.0140	6.6
Ni = 500 i = 1,2,3,4 Toi = 5.	.9749	.9718	.0035	.9745	.0030	.0119	33.0
Ni = 50 i = 1,2,3,4 Toi = 2.4 i = 1,2 Toi = 1.75 i = 3,4	.9754	.9632	.0151	.9826	.0122	.0114	1.48
Ni = 100 i = 1,2,3,4 Toi = 2.4 i = 1,2 Toi = 1.75 i = 3,4	.9789	.9706	.0107	.9822	.0089	.0079	2.96
Ni = 500 i = 1,2,3,4 Toi=2.4 i = 1,2 Toi =1.75 i = 3,4	.9818	.9781	.0042	.9822	.0039	.0049	14.8

TABLE I (Continued)

CASE III R_s = .9328

	$A_{(1-\alpha)}$	$\hat{R}_{s,L(\alpha)}$		$\hat{R}_s$		accuracy	TT
		mean	std.dev	mean	std.dev		
Ni = 50 i = 1,2,3,4 Toi = .5	.9376	.8686	.0534	.9441	.0450	.0048	1.71
Ni = 100 i = 1,2,3,4 Toi = .5	.9180	.8934	.0387	.9393	.0333	.0148	3.43
Ni = 500 i = 1,2,3,4 Toi = .5	.9382	.9250	.0158	.9408	.0145	.0054	17.1
Ni = 50 i = 1,2,3,4 Toi = 5.	.9256	.9076	.0191	.9253	.0176	.0072	17.1
Ni = 100 i = 1,2,3,4 Toi = 5.	.9257	.9144	.0127	.9252	.0122	.0071	34.3
Ni = 500 i = 1,2,3,4 Toi = 5.	.9248	.9202	.0054	.9248	.0052	.0080	171.5
Ni = 50 i = 1,2,3,4 Toi=2.4 i = 1,2 Toi =1.2 i = 3,4	.9297	.8976	.0357	.9349	.0300	.0031	4.72
Ni = 100 i = 1,2,3,4 Toi=2.4 i = 1,2 Toi=1.2 i = 3,4	.9281	.9077	.0248	.9321	.0220	.0047	9.44
Ni = 500 i = 1,2,3,4 Toi=2.4 i = 1,2 Toi=1.2 i = 3,4	.9315	.9229	.0099	.9315	.0094	.0013	47.2

TABLE II

ACCURACY OF COMPUTER SIMULATION FOR CASES IV, V, VI, AND VII

CASE IV $R_s = .9801$

	$A_{(1-\alpha)}$	$\hat{R}_{s,L(\alpha)}$ mean	$\hat{R}_{s,L(\alpha)}$ std.dev	$\hat{R}_s$ mean	$\hat{R}_s$ std.dev	accuracy	TT
Ni = 50 i = 1,2,3,4 Toi = .5	.9376	.9183	.0300	.9849	.0237	.0425	.5
Ni = 100 i = 1,21314 Toi = .5	.9683	.9494	.0205	.9855	.0160	.0118	1.0
Ni = 500 i = 1,2,3,4 Toi = .5	.9830	.9760	.0085	.9861	.0071	.0029	2.5
Ni = 50 i = 1,2,3,4 Toi = 1.0	.9683	.9423	.0248	.9800	.0196	.0118	1.0
Ni = 100 i = 1,2,3,4 Toi = 1.0	.9705	.9583	.0170	.9800	.0138	.0096	2.0
Ni = 500 i = 1,2,3,4 Toi = 1.0	.9797	.9738	.0064	.9808	.0057	.0004	5.0
Ni = 50 i = 1,2,3,4 Toi = 2.4	.9649	.9498	.0181	.9707	.0153	.0162	2.4
Ni = 100 i = 1,2,3,4 Toi = 2.4	.9675	.9579	.0124	.9709	.0109	.0126	4.8
Ni = 500 i = 1,2,3,4 Toi = 2.4	.9711	.9663	.0051	.9709	.0049	.0090	12.0

TABLE II (Continued)

CASE V R_s = .9250

	$A_{(1-\alpha)}$	$\hat{R}_{s,L(\alpha)}$		$\hat{R}_s$		accuracy	TT
		mean	std.dev	mean	std.dev		
Ni = 50 i = 1,2,3,4 Toi = 1.0	.9179	.8910	.0396	.9373	.0341	.0071	3.8
Ni = 100 i = 1,2,3,4 Toi = 1.0	.9239	.9045	.0274	.9338	.0244	.0011	7.6
Ni = 50 i = 1,2,3,4 Toi = 5.0	.9285	.9145	.0168	.9316	.0155	.0035	19.0
Ni = 100 i = 1,2,3,4 Toi = 5.0	.9315	.9217	.0121	.9322	.0177	.0065	38.0
Ni = 50 i = 1,2,3,4 Toi = 10.0	.9289	.9191	.0118	.9299	.0114	.0039	38.0
Ni = 100 i = 1,2,3,4 Toi = 10.0	.9283	.9209	.0088	.9283	.0084	.0033	76.0
Ni = 50 i = 1,2,3,4 Toi = 15.0	.9249	.9162	.0108	.9252	.0102	.0001	57.0
Ni = 100 i = 1,2,3,4 Toi = 15.0	.9250	.9190	.0075	.9252	.0072	.0000	114.0
Ni = 50 i = 1,2,3,4 Toi = 20.0	.9214	.9139	.0094	.9220	.0090	.0036	76.0
Ni = 100 i = 1,2,3,4 Toi = 20.0	.9215	.9159	.0069	.9215	.0066	.0035	152.0

TABLE II (Continued)

CASE VI R_s = .9400

	$A_{(1-\alpha)}$	$\hat{R}_{s,L(\alpha)}$		$\hat{R}_s$		accuracy	TT
		mean	std.dev	mean	std.dev		
Ni = 50 i = 1,2,3,4 Toi = 1.0	.9415	.9067	.0361	.9507	.0304	.0015	3.0
Ni = 100 i = 1,2,3,4 Toi = 1.0	.9462	.9235	.0250	.9506	.0218	.0062	6.0
Ni = 50 i = 1,2,3,4 Toi = 5.0	.9458	.9324	.0155	.9479	.0140	.0058	15.0
Ni = 100 i = 1,2,3,4 Toi = 5.0	.9462	.9369	.0105	.9469	.0102	.0062	30.0
Ni = 50 i = 1,2,3,4 Toi = 10.0	.9429	.9343	.0104	.9446	.0100	.0029	30.0
Ni = 100 i = 1,2,3,4 Toi = 10.0	.9461	.9385	.0081	.9450	.0077	.0061	60.0
Ni = 50 i = 1,2,3,4 Toi = 15.0	.9403	.9328	.0089	.9408	.0084	.0003	45.0
Ni = 100 i = 1,2,3,4 Toi = 15.0	.9416	.9359	.0066	.9415	.0063	.0016	90.0
Ni = 50 i = 1,2,3,4 Toi = 20.0	.9385	.9311	.0088	.9383	.0083	.0015	60.0
Ni = 100 i = 1,2,3,4 Toi = 20.0	.9385	.9339	.0057	.9389	.0055	.0015	120.0

TABLE II (Continued)

CASE VII $R_s = .8857$

	$A_{(1-\alpha)}$	$\hat{R}_{s,L(\alpha)}$ mean	$\hat{R}_{s,L(\alpha)}$ std.dev	$\hat{R}_s$ mean	$\hat{R}_s$ std.dev	accuracy	TT
Ni = 50 i = 1,2,3,4 Toi = 1.2	.8894	.8479	.0418	.8942	.0379	.0037	7.04
Ni = 100 i = 1,2,3,4 Toi = 1.2	.8910	.8635	.0307	.8934	.0284	.0053	14.08
Ni = 500 i = 1,2,3,4 Toi = 1.2	.8913	.8831	.0025	.8939	.0120	.0056	35.20

CHAPTER V

SUMMARY AND CONCLUSIONS

A partial explanation of the procedure developed in reference 1., which is used to compute an upper limit on failure rate estimates and lower limits on reliability estimates, has been presented in Chapter II. A simulation of this procedure, without the assumption of an exponential failure rate, was proposed in Chapter III and the results tabulated in Tables I and II.

Examination of the results of the first three cases shows that the average accuracy, $\overline{D}$, for the various planned test times, PTT, are as follows:

PTT	$\overline{D}$
.5	.0174
5.	.0147
ad hoc	.0047

Many more simulations would have to be conducted, using various parameters of the failure rate distributions, in order to establish final results. But from the cases presented it would seem that the test time selected from the ad hoc rule stated in Chapter III, produced the best accuracy regardless of sample size. From Cases IV and VII the average accuracy using the ad hoc rule for test times was .0085. In Cases V and VI test times of 1.0, 5.0, 10.0, 15.0, and 20.0 mission units were used and the accuracy varied from a low of .0000 to a high of .0071.

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APPENDIX A

A. Gamma Distribution

$$f(t) = \frac{s^r \, t^{r-1} \, e^{-st}}{\Gamma r} \tag{9}$$

$$Z(t) = \frac{f(t)}{R(t)}$$

$$Z(t) = \frac{s^r \, t^{r-1} \, e^{-st}}{e^{-st} \sum_{i=0}^{r-1} \frac{(st)^i}{i}} \tag{10}$$

To evaluate the parameters and meet the requirements that $R_i = .995$ the following procedure was used:

$$R(t) = P[T > t]$$

$$R_i = R(1)$$

$$R_i = P[T > 1]$$

$$R_i = P[2sT > 2s]$$

$$R_i = P[\chi^2_{2r} > 2s]$$

$$.995 = P[\chi^2_{2r} > 2s]$$

Therefore for a given value of the parameter r, and the component reliability, a chi-square table look up would give the value of the second parameter s. For $r = 1.5$, $s = .036$. The specific value of r was chosen because the failure rate curve is representative of those curves used by contractors. A plot of equation (10) with the selected parameters is shown in Figures IA through IIIA.

B. Truncated Normal Distribution

$$f(t) = \frac{b}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} = b\phi(u) \qquad (11)$$

where $u = \frac{t-\mu}{\sigma}$

$$b = \frac{1}{\Phi\left(\frac{\mu}{\sigma}\right)} \qquad (12)$$

$$Z(t) = \frac{f(t)}{R(t)}$$

$$Z(t) = \frac{\frac{b}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}}{\frac{b}{\sigma\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} dt} \qquad (13)$$

$$Z(u) = \frac{\frac{1}{\sigma}\ \phi(u)}{1 - \Phi(u)}$$

For Case I, R_i was specified at .995 and the following procedure was used to evaluate the parameters σ and μ.

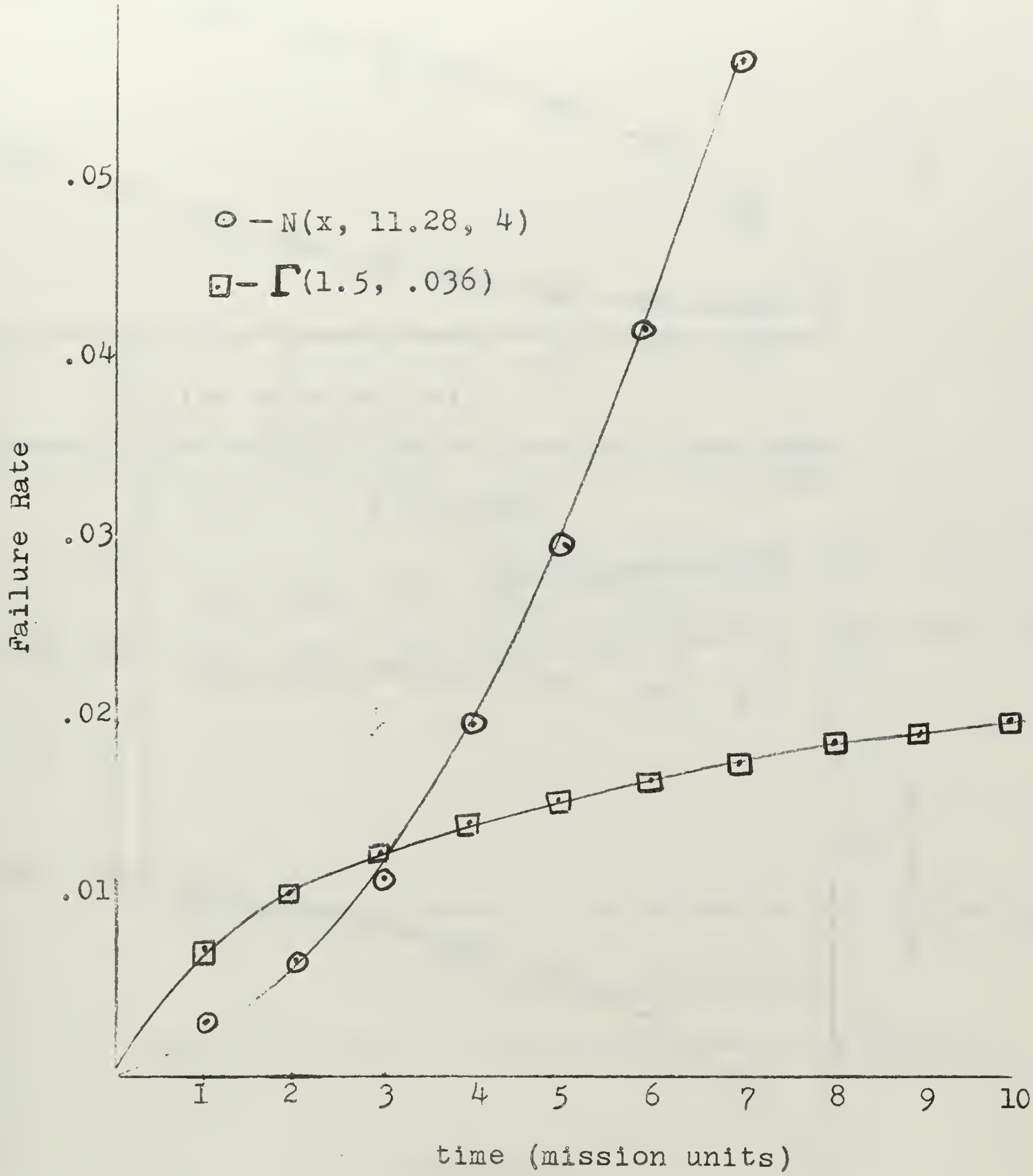
$$R_i = P(T > 1)$$

$$R_i = P\left(\frac{T-\mu}{\sigma} > \frac{1-\mu}{\sigma}\right)$$

$$R_i = b\left(1 - \Phi\left(\frac{1-\mu}{\sigma}\right)\right)$$

$$.995 = b\left(\Phi\left(-\frac{1-\mu}{\sigma}\right)\right)$$

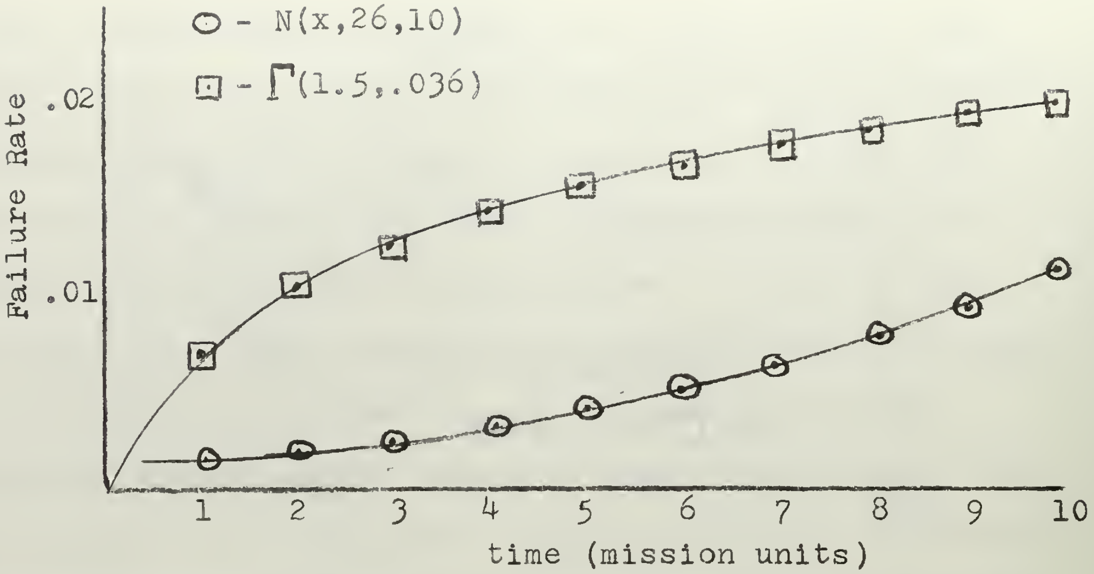
In Case I, σ and μ were selected to be 4 and 11.28. From equation (12), b = 1.0024. For Cases II and III the values of the parameters were chosen such that the failure rate curves would be flatter and again better approximate the failure rate curves used by contractors. In Case III the restriction of a lower component reliability was also placed on the parameters. Parameters for Cases V and VI were selected as miscellaneous values in order to give more complete results to the analysis. Plots of equation (13) with the selected parameters are shown in Figure IA through IVA.



Gamma and truncated normal failure rate curves for Case I.

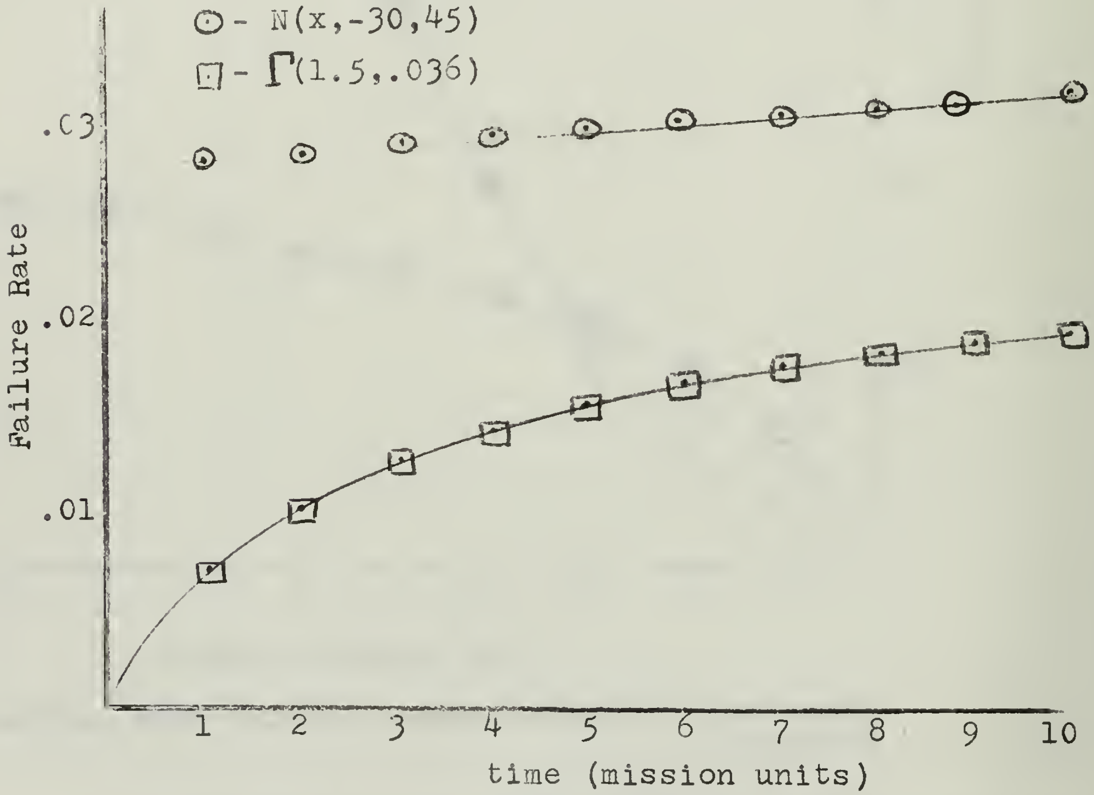
Gamma failure rate curve for Case IV.

FIGURE IA



Gamma and truncated normal failure rate curves for Case II.

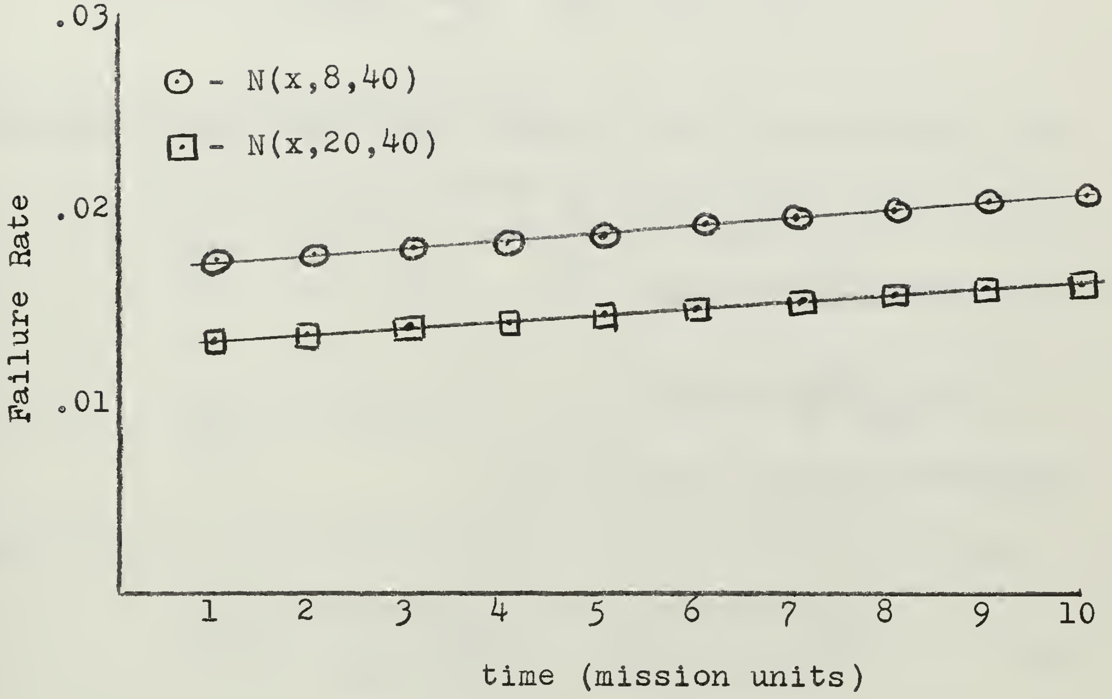
FIGURE II A



Gamma and truncated normal failure rate curves for Case III.

Truncated normal failure rate curve for Case VII.

FIGURE III A



Truncated normal failure rate curves for Cases V and VI.

FIGURE IV A

APPENDIX B

COMPUTATION OF AD HOC TEST TIMES

The test times for Cases I, II, and III were selected as follows:

1. .5 mission units
2. 5. mission units
3. T_{oi} such that $-\frac{1}{T_{oi}} \ln(R(T_{oi})) = Z(1)$ (7)

The computation of the planned test times under the ad hoc rule for the distributions used is given below.

A. Gamma Distribution

$$R(t) = P(T > t)$$

substituting into equation (7)

$$-\frac{1}{T_{oi}} \ln(R(T_{oi})) = -\frac{1}{T_{oi}} \ln(P(T > T_{oi}))$$

$$= -\frac{1}{T_{oi}} \ln(P(2sT > 2sT_{oi}))$$

$$-\frac{1}{T_{oi}} \ln(R(T_{oi})) = -\frac{1}{T_{oi}} \ln(P(\chi^2_{2r} > 2sT_{oi}))$$

Therefore the following equation must be solved for T_{oi}:

$$-\frac{1}{T_{oi}} \ln(P(\chi^2_{2r} > 2sT_{oi})) = Z(1)$$

Solving equation (10) with the parameters $r = 1.5$ and $s = .036$, $Z(1) = .0076$. Substituting into the above equation:

$$- \ln (P(\chi_3^2 > .072\ T_{oi})) = .0076\ T_{oi}$$

Solving gives $T_{oi} = 2.4$ mission units.

B. Truncated Normal Distribution

$$R(t) = P(T > t)$$

substituting into equation (8):

$$- \frac{1}{T_{oi}} \ln(R(T_{oi})) = - \frac{1}{T_{oi}} \ln (P(T > T_{oi}))$$

$$- \frac{1}{T_{oi}} \ln (R(T_{oi})) = - \frac{1}{T_{oi}} \ln(b(1 - \Phi(\frac{T_{oi} - \mu}{\sigma}))$$

Therefore the following equation must be solved for T_{oi}:

$$- \ln (b(1 - \Phi(\frac{T_{oi} - \mu}{\sigma}))) = Z(1)\ T_{oi} \qquad (14)$$

Solving equation (13) with the parameters for Case I, $Z(1) = .0037$. Substituting into the above equation:

$$- \ln (1.0024\ (1 - \Phi(\frac{T_{oi} - 11.28}{4}))) = .0037\ T_{oi}$$

Therefore for the truncated normal distribution in Case I, $T_{oi} = 1.8$ mission units. Solving equation (14) with the parameters for Cases II and III, the following times result:

Case II T_{oi} = 1.75 mission units

Case III T_{oi} = 1.2 mission units

APPENDIX C

The gamma and truncated normal failure times were computed using a Monte Carlo method. A pseudo uniform random number generator was used to generate a normal (0,1) random variate, V. The random number generator and the compiling of the normal variate was programmed in machine language and was used as a subroutine function called SNORM. The random number generator used is called URN and is available at the N.P.G.S. computer facility library.

The gamma distributed failure time is compiled from a chi-square value with 2r degrees of freedom and a parameter, s, such that the gamma variate $= \frac{\chi^2_{2r}}{2s}$. Since χ^2_{2r} is equal to the sum of 2r squared normal (0,1) variates, the gamma distributed time is compiled in the computer simulation by the value, $\frac{\sum_{i=1}^{2r} V^2}{2s}$. The normal distributed failure times are truncated at zero and they are computed from the generated normal variates. They are set equal to $\sigma V + \mu$, where σ and μ are the parameters of the distribution.

The general flow of the simulation is to test each component seperately for N_i replications. The estimate of that component failure rate, $\hat{\lambda}_i$, is given by statement number 586, which represents equation (1). After each component has been tested N_i times and it's failure rate estimated, the system failure rate is given by $\hat{\lambda}_s$. The point estimate of system reliability is then computed in statement 591, which represents equation (2). The number of failures and the standard deviations for the trials of all four components are compiled in statements 587, and 588. The resulting formulae for the lower confidence limits of the failure rates are given in statements 616 and 624 depending on the number of failures, as explained in equations (4a) and (4b). The whole process is repeated five hundred times and the distribution ordered. The mean and standard deviation for $\hat{R}_{s,L(\alpha)}$ and $\hat{R}_s$, the reliability point estimate, are also calculated in the computer simulation.

```
      DIMENSION A(3),BETA(31),      R(4),RLHAT(500),
     *RSHAT(500),RSBAR(3),SDEV(3),SI(4),SIGMA(4),
     *TT(4,3),XMEAN(4),RLBAR(3),RSDEV(3)
      DATA BETA/1.507,1.369,1.309,1.272,1.246,1.227,
     *1.212,1.200,1.190,1.181,1.173,1.167,1.161,
     *1.156,1.153,1.146,1.144,1.139,1.135,1.132,
     *1.129,1.126,1.124,1.122,1.119,1.117,1.115,
     *1.113,1.111,1.110,1.0/
C
C     READ IN SELECTED MISSION TIMES TT.
C     READ IN SAMPLE SIZE JD.
C     READ IN PARAMETERS FOR GAMMA DISTRIBUTION,R,SI.
C     READ IN PARAMETERS FOR TR. NORMAL DIST. SIGMA,XMEAN.
C     BETA CORRECTION FACTOR IS READ IN AS DATA
C
      JD=50
      READ 3, ((TT(I,J),I=1,4),J=1,3)
      READ 3,  (R(I),I=1,4)
      READ 3,(SI(I),I=1,4)
      READ 3,(SIGMA(I),I=1,4)
      READ 3,(XMEAN(I),I=1,4)
    3 FORMAT(4F10.3)
C
C     M IS INDEX FOR 500 CALCULATIONS OF RSL .
C     I IS INDEX FOR 4 COMPONENTS IN SERIES.
C     J IS INDEX FOR NUMBER IN SAMPLE SIZE.
C     K IS INDEX FOR TEST TIMES.
C
      DO 9000 K=1,3
      RLDUM=0.0
      RSDUM=0.0
C
C     GENERATE 500 VALUES OF RSL.
C
      DO 650 M=1,500
      TLAM=0.0
      DUM=0.0
      CHAT=0.0
      FTOTE=0.0
      DO 590 I=1,4
      F=0.0
      SUM=0.0
      DO 585 J=1,JD
C
C     IF COMPONENT I HAS GAMMA DISTRIBUTION, GO TO
C     STATEMENT NUMBER 300 FOR GENERATION OF TIMES.
C
C     IF COMPONENT I HAS TRUNCATED NORMAL DISTRIBUTION
C     GO TO STATEMENT 400 FOR GENERATION OF TIMES.
C
C     IN THIS PROGRAM COMPONENTS 1 AND 2 HAVE GAMMA
C     FAILURE DISTRIBUTIONS AND COMPONENTS 3 AND 4
C     HAVE TRUNCATED NORMAL FAILURE DISTRIBUTIONS.
C
      IF (I-2) 300,300,400
  300 IA=2.*R(1)
      TOTAL=0.0
      DO 330 IB=1,IA
      V=SNORM(1)
  330 TOTAL=TOTAL+(V*V)
      TP=TOTAL/(2.*SI(1))
      GO TO 500
  400 CONTINUE
  402 V=SNORM(1)
      TEST=SIGMA(4)*V+XMEAN(4)
      IF(TEST-0.0)402,402,405
  405 TP=TEST
  500 IF(TP-TT(I,K))510,550,550
  510 T=TP
      F=F+1.0
      GO TO 560
  550 T=TT(I,K)
  560 SUM=SUM+T
  585 CONTINUE
      XN=JD
  586 HLAM=(F/SUM)*(2.*XN/(2.*XN+1.))
      TLAM=TLAM+HLAM
  587 FTOTE=FTOTE+F
  588 CHAT=CHAT+HLAM/SUM
      DUM=DUM+1.0/SUM
  590 CONTINUE
  591 RSHAT(M)=EXP(-TLAM)
      NTOTE=FTOTE+1.0
      IF(NTOTE-31)614,614,613
  613 NTOTE=31
  614 CONTINUE
      IF(FTOTE-0.0) 615,615,620
  615 TDUM=BETA(NTOTE)*.842
  616 ULAM=TDUM*TDUM*DUM/4.0
      GO TO 625
  620 CHAT=CHAT/TLAM
      TDUM= BETA(NTOTE)*.842
      FDUM=SQRT(4.*TLAM*TDUM*TDUM*CHAT+TDUM*TDUM*
     *TDUM*TDUM*CHAT*CHAT)
  624 ULAM=(2.*TLAM+TDUM*TDUM*CHAT+FDUM)/2.0
  625 RLHAT(M)=EXP(-ULAM)
      RLDUM=RLDUM+RLHAT(M)
      RSDUM=RSDUM+RSHAT(M)
  650 CONTINUE
C
C     COMPUTATION FOR MEAN AND STANDARD DEVIATION OF THE
C     DISTRIBUTION FOR RSL .
C
      RSBAR(K)=RSDUM/500.
      RLBAR(K)=RLDUM/500.
      SLDUM=0.0
      SDUM=0.0
      DO 690 II=1,500
      SLDUM=SLDUM+(RLHAT(II)-RLBAR(K))**2
  690 SDUM=SDUM+(RSHAT(II)-RSBAR(K))**2
      SDEV(K)=SQRT(SLDUM/500.)
      RSDEV(K)=SQRT(SDUM/500.)
C
C
C     STATEMENTS 700 TO 705 ARE USED FOR ORDERING THE
C     500 CALCULATED VALUES OF RSL  AND SELECTING THE
C     400 TH VALUE OF THE DISTRIBUTION.
C
  700 DO 705 NX=1,101
      B=RLHAT(NX)
      NA=NX+1
      DO 706 NB=NA,500
      IF(B-RLHAT(NB))703,706,706
  703 TEMP=RLHAT(NB)
      RLHAT(NB)=B
      B=TEMP
  706 CONTINUE
  705 CONTINUE
      A(K)=B
C
C     OUTPUT STATEMENTS
C
      WRITE(6,1) JD
    1 FORMAT(1H1,33X,'COMPONENT TEST TIMES FOR 'I3' RE
     *PLICATIONS')
      WRITE (6,2)(I,TT(I,K),I=1,4)
    2 FORMAT(/,46X,'COMPONENT 'I2,' TIME ='F8.3)
      WRITE(6,902) A(K),SDEV(K),RLBAR(K)
  902 FORMAT(//,20X,'A='F10.4,5X,'SDEV=',F10.4,5X,'RSL
     *BAR='F10.4)
      WRITE(6,904) RSBAR(K),RSDEV(K)
  904 FORMAT(//,30X,'RBAR = 'F10.4,5X,'SDEV ='F10.4)
 9000 CONTINUE
      END
```

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13. ABSTRACT

The accuracy of the lower confidence limit procedure in NAVWEPS OD 29304 is analyzed when the failure distributions of the components in the series system are either gamma or truncated normal. Several representations of the accuracy are supplied.

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
System Reliability						
Reliability						
Gamma Failure Distribution						
Truncated Normal Failure Distribution						
Reliability Confidence Limit						

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